

# Analysis and Control of Collective Dynamics in Brain Networks of Oscillators

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# Synchronization in the Brain

Synchronization phenomena are ubiquitous in neural systems

*in health...*

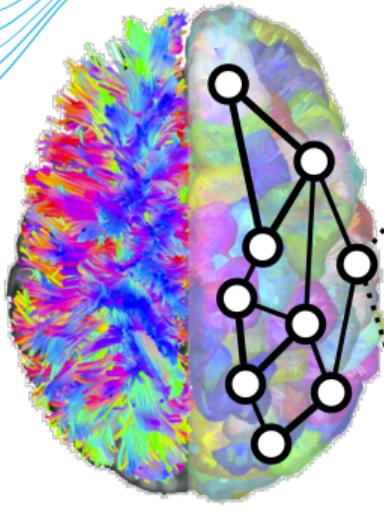
- ▷ information processing
- ▷ memory storage/retrieval
- ▷ motor coordination
- ▷ circadian rhythms

*...and disease*

- ▷ dementias (Parkinson's)
- ▷ cognitive decline
- ▷ epilepsy and seizures
- ▷ neurological damage

Characterization and control of synchronization are of paramount importance... **how to do it?**

# Brain Networks of Oscillators



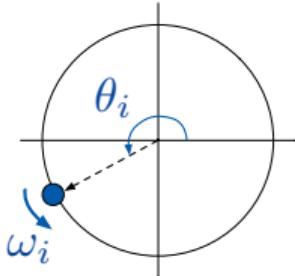
connectivity      parcellation

brain network

- ▷ nodes  $\leftarrow$  brain regions
- ▷ edges  $\leftarrow$  axonal bundles

## Kuramoto Oscillator

$$\dot{\theta}_i(t) = \omega_i + \sum_{j \neq i} a_{ij} \sin(\theta_j(t) - \theta_i(t))$$



- ▷  $\theta_i$ : phase
- ▷  $\omega_i$ : natural frequency
- ▷  $a_{ij}$ : coupling strength

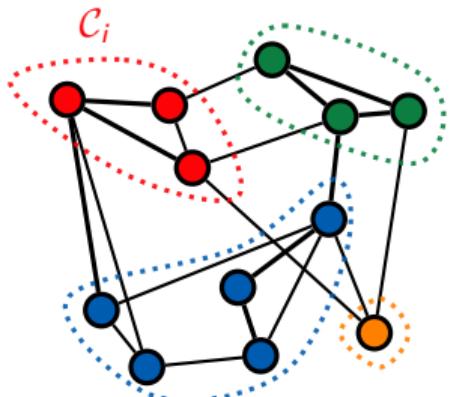
[Cabral *et al.*, NeuroImage 2011]

each Kuramoto oscillator in the brain network represents a neural population (brain region)

↓  
phase trajectories represent neural activity

↓  
**synchronization**  
analysis of coupled oscillators!

# Cluster Synchronization



for a network partition  $\mathcal{P} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$ :  
phases of the oscillators belonging to  
the same cluster  $\mathcal{C}_i$  evolve synchronized

## Cluster Synchronization Manifold

$$\mathcal{S}_{\mathcal{P}} = \{\theta \mid \theta_i = \theta_j \text{ for all } i, j \in \mathcal{C}_k, k = 1, \dots, m\}$$

$$\dot{\theta}_i = \omega_i + \sum_{j \neq i} a_{ij} \sin(\theta_j - \theta_i)$$

**invariance** of  $\mathcal{S}_{\mathcal{P}}$  iff:

- (a)  $\omega_i = \omega_j$  in the same cluster
- (b) balanced inter-cluster couplings:

$$\sum_{k \in \mathcal{C}_\ell} a_{ik} - a_{jk} = 0 \text{ for every } i, j \in \mathcal{C}_z$$

and  $z, \ell \in \{1, \dots, m\}$ , with  $z \neq \ell$

**stability** of  $\mathcal{S}_{\mathcal{P}}$  if:

intra-cluster couplings  $\gg$  inter-cluster couplings

**stability** of  $\mathcal{S}_{\mathcal{P}}$  if:

heterogeneous natural frequencies across clusters

# Control of Cluster Synchronization

damage or neurological disorders cause *abnormal* cluster synchronization

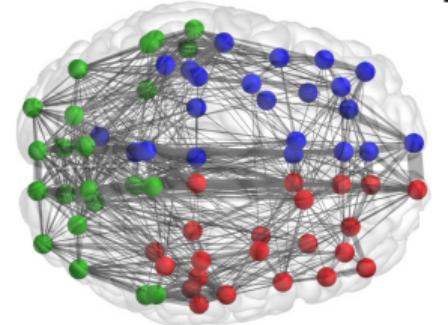


design of sparse optimal corrections to structure ( $a_{ij}$ ) and frequencies ( $\omega_i$ )

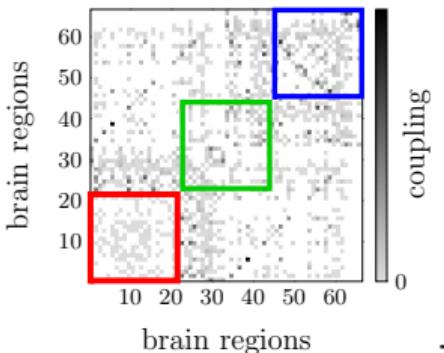


**desirable** cluster synchronization

[Menara et al., IEEE CDC 2019]



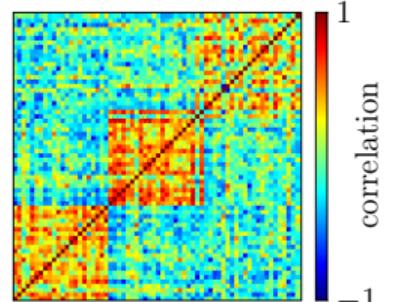
$$A = [a_{ij}]$$



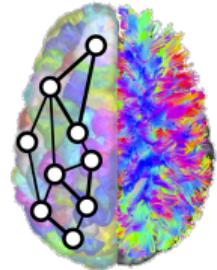
$$A + \Delta, \omega + \mu$$

minimally invasive structural and frequency corrections

desired cluster synch.  
(correlation of oscillatory neural activity time series)



# Conclusion and References



## In this work:

- ▷ **Synchronization** phenomena are ubiquitous in the brain
- ▷ **Characterization** of cluster synchronization in brain networks of oscillators
- ▷ **Control** of cluster synchronization with localized minimally invasive interventions

## References:

[J. Cabral *et al.*, NeuroImage 2011], [T. Menara *et al.*, ACC 2019], [T. Menara *et al.*, IEEE CDC 2019], [T. Menara *et al.*, IEEE TCNS 2020]



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