From Cluster Synchronization of Oscillators to Functional Connectivity

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Objectives

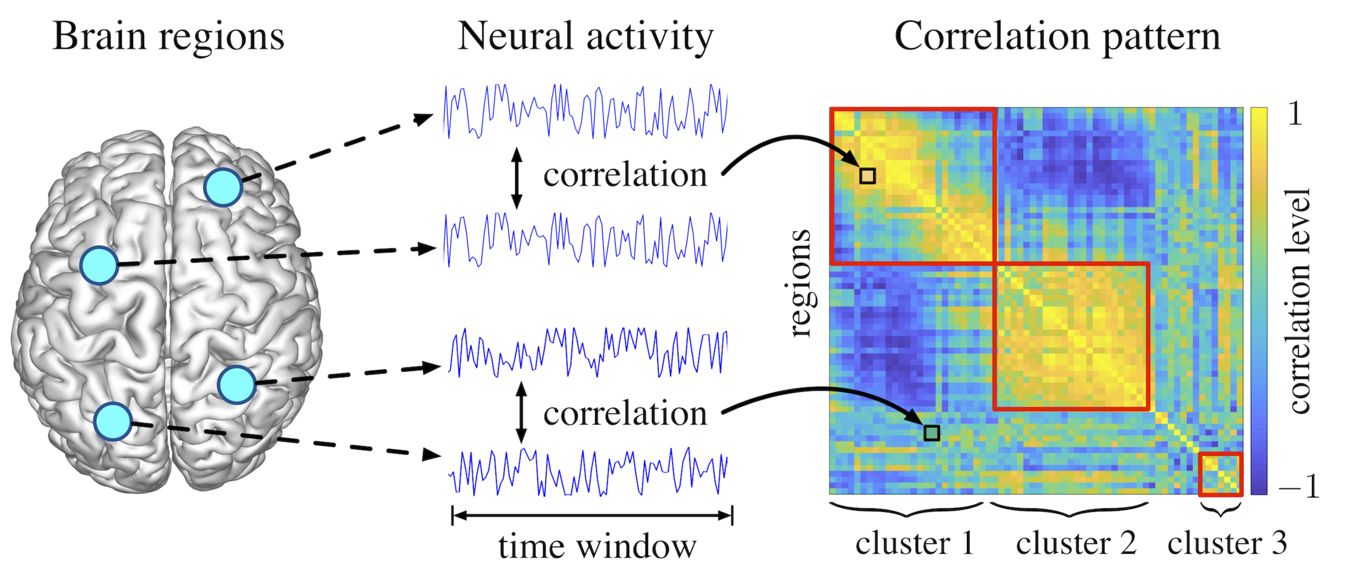
1. Analyze functional connectivity by means of analytical tools [1].
2. Apply theory to real data by deriving approximate conditions that closely capture the onset of cluster synchronization [2].

Abstract

Synchronization in the brain is a puzzling, yet ubiquitous, phenomenon. For instance, *synchronization patterns* of neural activity are thought to be biomarkers of neurological disorders, and synchronized regions correlate with a number of cognitive states. In this work, we shed light on the mechanisms underlying synchronization patterns of oscillatory neural activity. Specifically, we study cluster synchronization in networks of oscillators with Kuramoto dynamics, where multiple groups of oscillators with identical phases coexist in a connected network. Each oscillator models the activity of a brain region, and for arbitrary configurations we derive conditions on the oscillators' natural frequencies and interconnection weights to enable the onset of cluster synchronization.

Model Motivation

Functional connectivity in the human brain stems from synchronization of oscillatory neural activity in different brain regions.



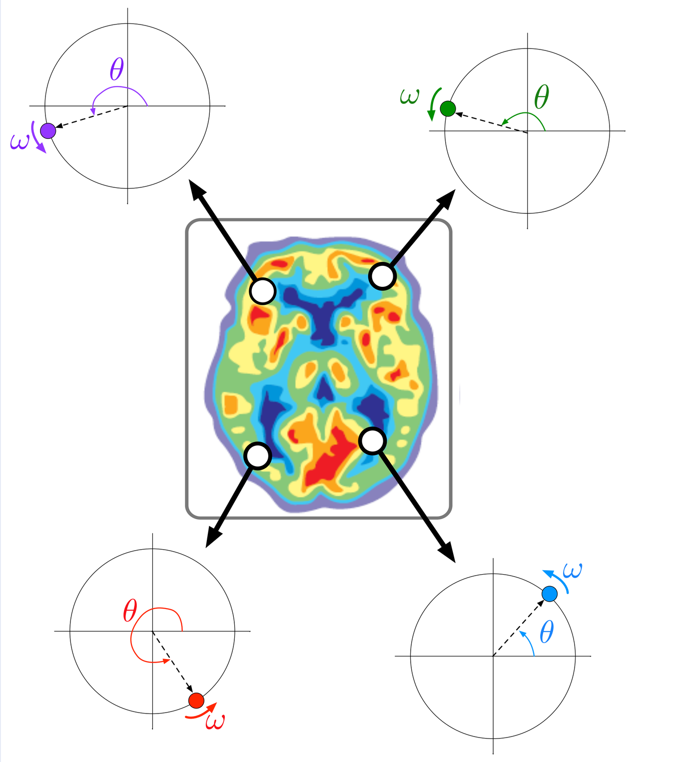
We assume that at each node of the structural brain network exists a community of excitatory and inhibitory neurons whose dynamical state is in a regime of self-sustained oscillations.

Sparsely connected heterogeneous Kuramoto oscillators

$$\dot{\theta}_i = \omega_i + \sum_{j \neq i} a_{ij} \sin(\theta_j - \theta_i)$$

θ_i is the phase of the i-th oscillator, ω_i is its natural frequency, and a_{ij} denotes the coupling strength between the j-th and the i-th oscillators.

Problem Setup



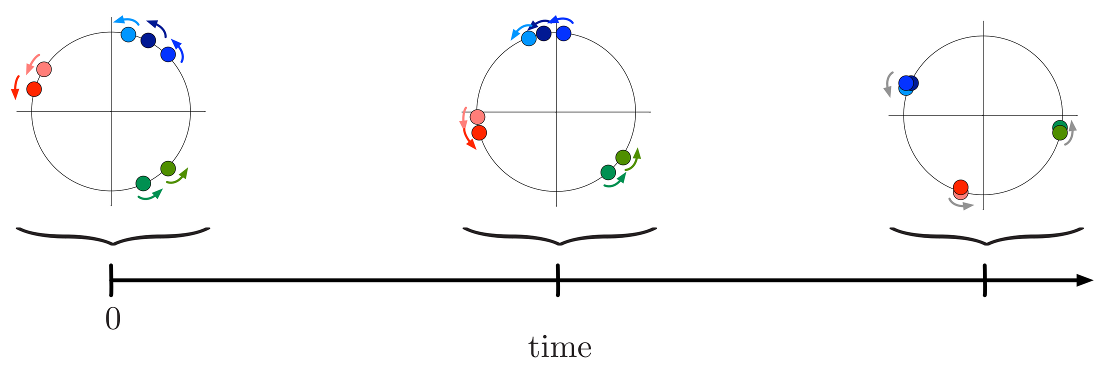
The brain can be abstracted by means of a network composed of nodes and edges:

- **structural brain networks**
  - nodes ⟵ brain regions
  - edges ⟵ white matter tracts that connect the regions
- **functional brain networks**
  - nodes ⟵ brain regions
  - edges ⟵ level of correlation between neurophysiological signals (e.g. fMRI)

Given a partition of the nodes $\mathcal{P} = \{\mathcal{P}_1, \dots, \mathcal{P}_m\}$, we derive conditions that guarantee exponential stability of the *cluster synchronization manifold*

$$\mathcal{S}_{\mathcal{P}} = \{\theta \;:\; \theta_i = \theta_j \text{ for all } i, j \in \mathcal{P}_k,\; k = 1, \dots, m\}$$

stable manifold $\mathcal{S}_{\mathcal{P}}$, $\mathcal{P} = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$:



Analytical Results

Result based on perturbation theory of dynamical systems:

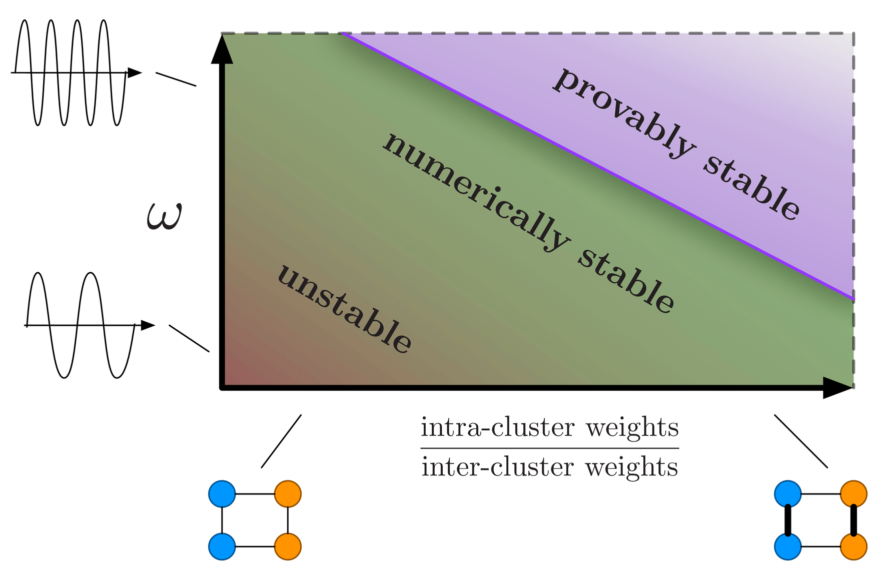
Theorem 1: sufficient condition on the network weights

The cluster synchronization manifold $\mathcal{S}_{\mathcal{P}}$ is stable if: $\dfrac{\text{intra-cluster couplings}}{\text{inter-cluster couplings}} \gg 1$

Result based on Lyapunov's stability theory:

Theorem 2: sufficient conditions on the natural frequencies

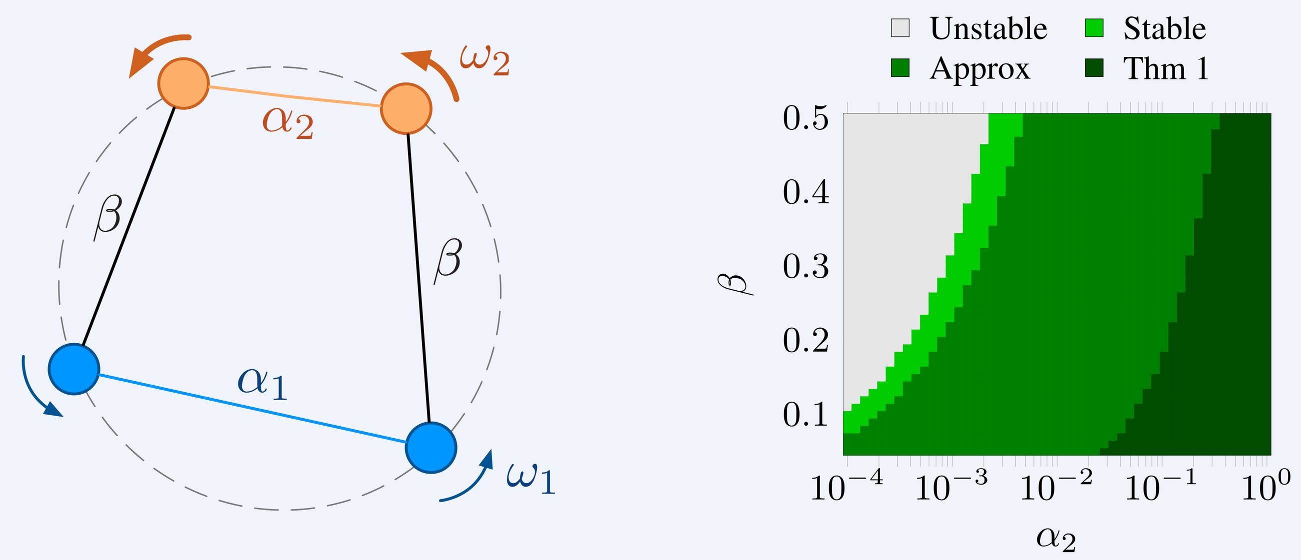
The synchronization manifold $\mathcal{S}_{\mathcal{P}}$ is stable if the frequencies across different clusters are sufficiently different



Tighter Approximate Results

Linearization of the network dynamics around cluster-synchronized trajectories + small gain theorem = **tight stability conditions**.

Example



(a) (b)

α_1, α_2 ⟵ intra-cluster coupling, β ⟵ inter-cluster coupling

We fix $\alpha_1 = \omega_1 = 1$, $\omega_2 = 8$, and let β, α_2 vary as indicated. Fig. 1(a) is a network of 4 Kuramoto oscillators. Fig. 1(b) shows that the approximate stability condition is much tighter than the condition in Theorem 1. Our approximate conditions are able to tightly capture the onset of cluster synchronization.

Conclusion and Future Work

- Exact stability conditions: the cluster synchronization manifold is stable when the intra-cluster weights are sufficiently larger than the inter-cluster weights, and also when the natural frequencies across different clusters are sufficiently heterogeneous.
- Approximate stability conditions: combine the independent mechanisms of stability based on the network weights and oscillators' natural frequencies, and are shown to be considerably more accurate than our exact conditions.
- Future work: validation of the clustering mechanisms on different datasets, prediction of clusters from connectivity information, data-driven control of cluster synchronization.

References

[1] **T. Menara**, G. Baggio, D. S. Bassett, and F. Pasqualetti. Stability conditions for cluster synchronization in networks of heterogeneous Kuramoto oscillators. *IEEE Transactions on Control of Network Systems*, 2019.

[2] **T. Menara**, G. Baggio, D. S. Bassett, and F. Pasqualetti. Exact and approximate stability conditions for cluster synchronization of Kuramoto oscillators. In *American Control Conference*, Philadelphia, PA, USA, 2019.